Planar Bi Rotor Helicopter

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I. Abstract

In the first step, a linear control model is designed and tested under model mismatch and adaptive control scenarios. The objective is to show that an adaptive control system can better meet the performance specifications compared to a static controller. The second step involves designing a nonlinear controller that leads to a last step consisting in augmenting this non linear control structure for adaptation.

II. Introduction

The problem of controlling a planar Bi Rotor Helicopter is modeled with the following equations :

$$m\ddot{q} = -d\dot{q} + \begin{pmatrix} -sin(\theta) \\ cos(\theta) \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} \vec{f} - m\vec{g}$$

$$J\ddot{\theta} = r_a \begin{pmatrix} 1 & 1 \end{pmatrix} \vec{f}$$

Where q contains the position of the helicopter and θ is the orientation. Defined parameters value are :

m	6 kg
d	0.1 kg/sec
g	9.8 m/sec
r_a	0.25 m
J	0.1425 kg.m^2

Let's consider the state,

$$X = \begin{bmatrix} x & y & \theta & \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T$$

which leads to the following state space representation :

$$\dot{X} = f(X, \vec{g}, u)$$

Where the command u are the two independent fans' thrust

$$u = \begin{pmatrix} f1 \\ f2 \end{pmatrix}$$

We will evaluate performances of tracking a desired trajectory called r and point to point movement.

$$r = \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix}, \dot{r} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

or an amplified Lemniscate of Gerono trajectory defined as

$$r = \begin{bmatrix} A\cos(2\pi f * t) \\ A\sin(2\pi f * 2t)/2 \end{bmatrix}$$

We will experiment with various amplitudes and frequencies and analyze the behavior of the birotor under different controllers.

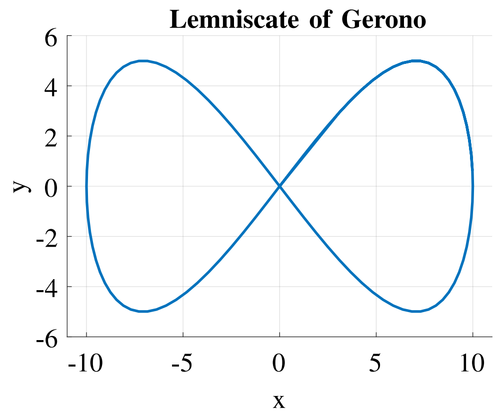


Fig. 1: Lemniscate of Gerono trajectory for an amplitude of $A = 10$ and a period of $T = 100s$

III. Step 1 : A Linear Controller

As a first approach, we can linerize the system about $\theta = 0$ and define two matrices Ae and Be linearized from the actual dynamics A and B.

$$Ae = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -g & -\frac{d}{m} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{d}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} Be = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m} & \frac{1}{m} \\ \frac{r}{J} & -\frac{r}{J} \end{bmatrix}$$

The following will study a basic LQR linear controller and the augmentation of this controller to an adaptive linear controller.

A. LQR controller

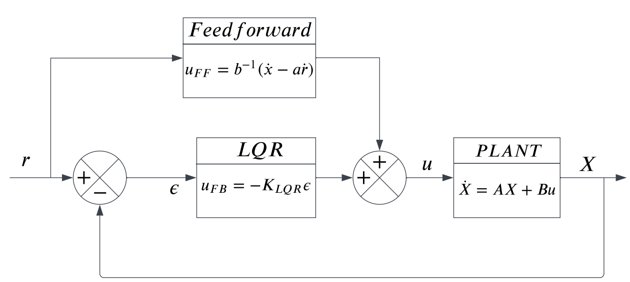


Fig. 2: LQR Controller with feedforward term a and b being the terms in A and B influencing $\dot{X}$

1) Static linear controller with matched parameters: In general, the performances desired for the system can be expressed through a performance matrix Q. In this case, the desired performance is a trade-off between tracking the input signal and minimizing the control effort. To reflect this, we have chosen a matrix Q with high values on the diagonal for the positions state variables that we want to track accurately. Lower values for the speed variables for which we want to minimize the control effort. We have chosen to track accurately x and y.

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Applying this linear controller to the plant results in the following responses.

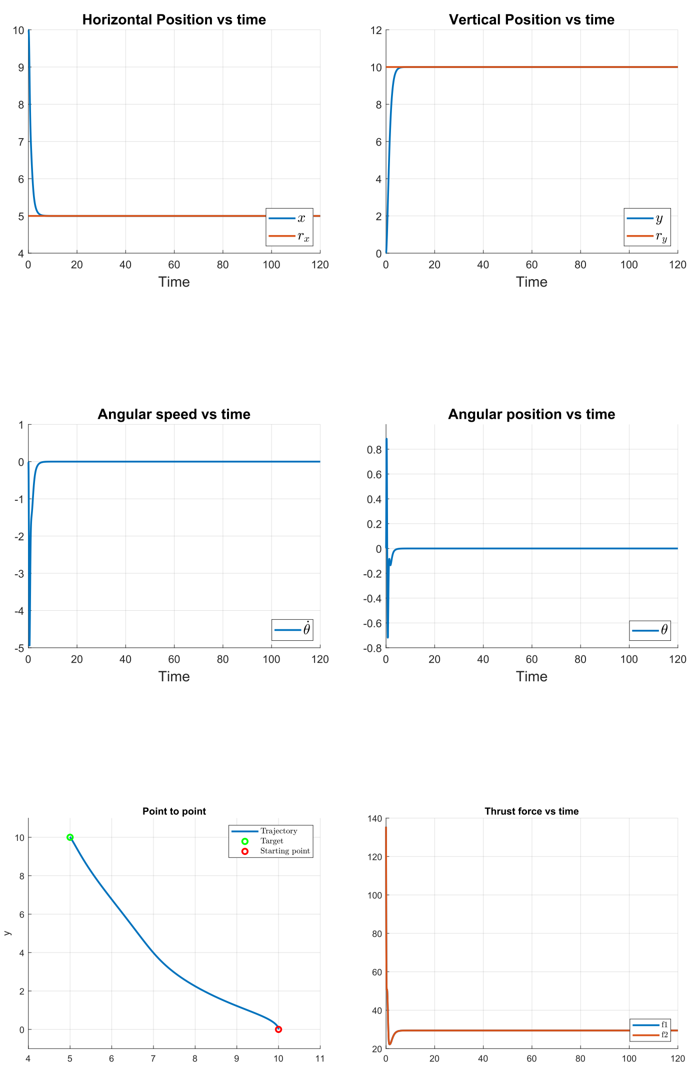


Fig. 3: Plant response for a point to point input using a LQR linear controller. Tracking is achieved with no static error. The limitation on the angle θ causes a limitation on the helicopter maximum horizontal speed.

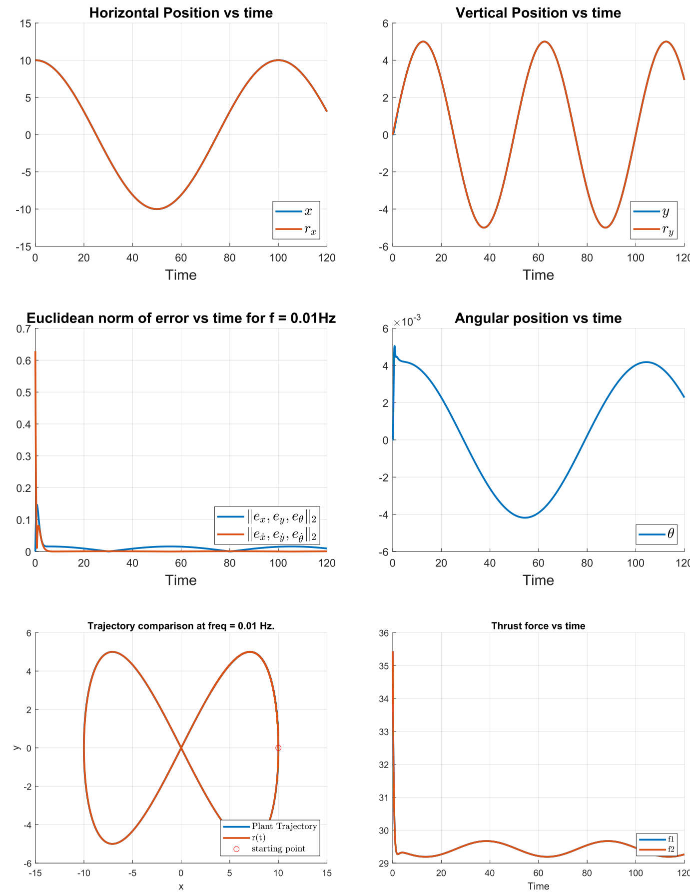
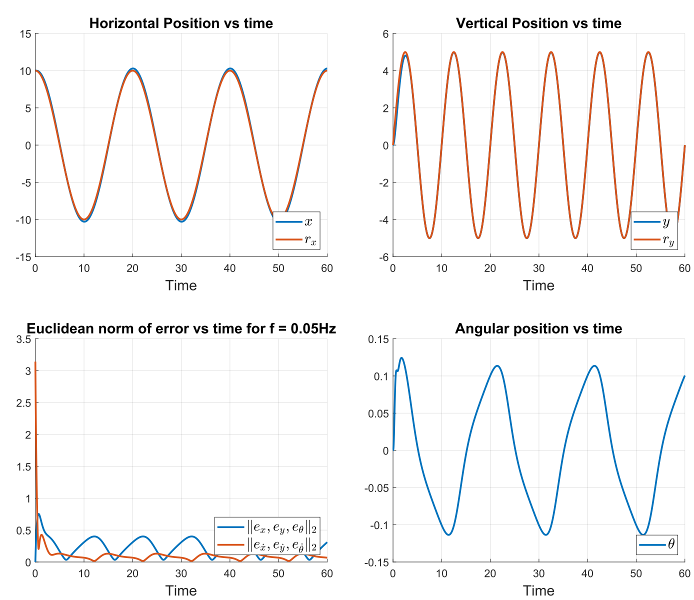


Fig. 4: Error based regulation for a Lemniscate of Gerono trajectory of amplitude 5 and period of 100 s

However, as we increase the frequency and demand greater dynamism and responsiveness from the helicopter, the linear controller becomes insufficient and reaches its limitations. As a result, we present below several different results obtained while increasing the frequency:



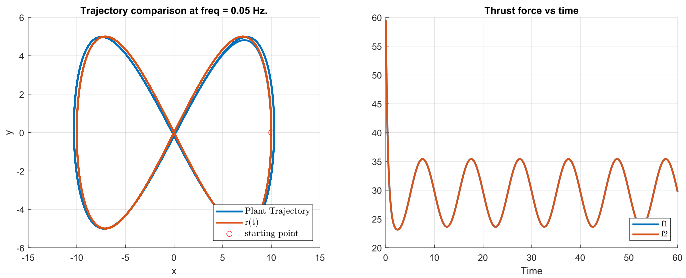


Fig. 5: Plant regulation for a Lemniscate of Gerono trajectory of amplitude 5 and period of 20 s

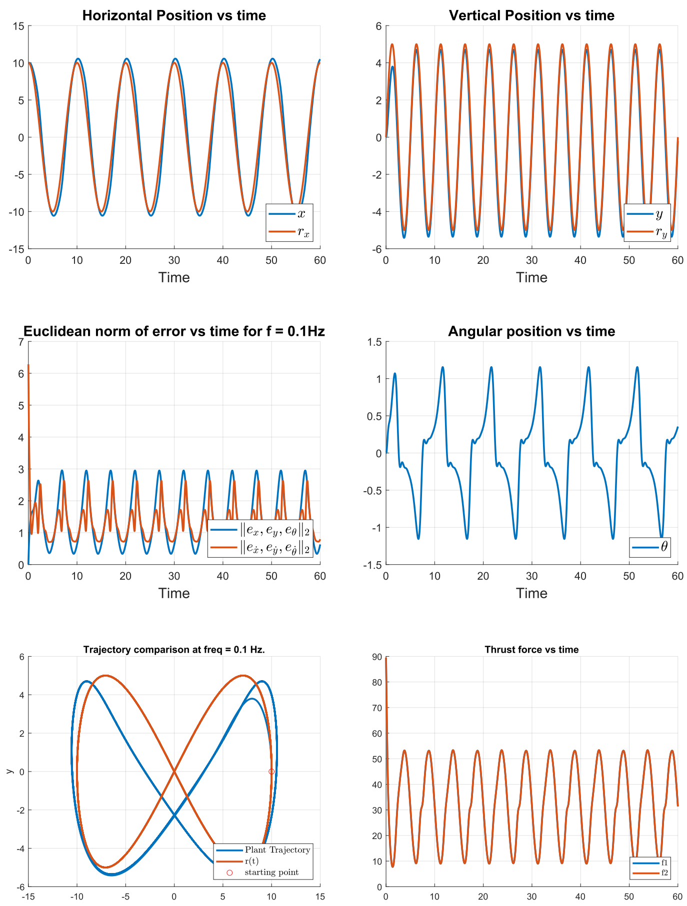


Fig. 6: Plant regulation for a Lemniscate of Gerono trajectory of amplitude 5 and period of 10 s

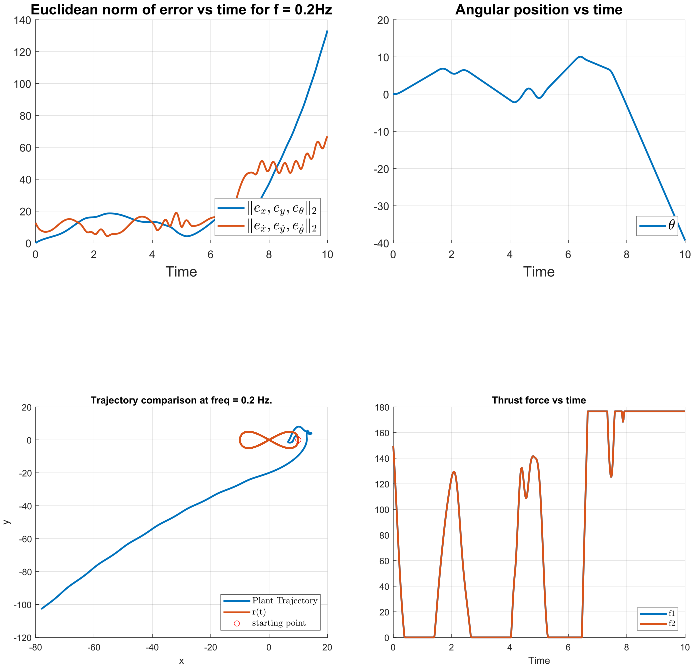


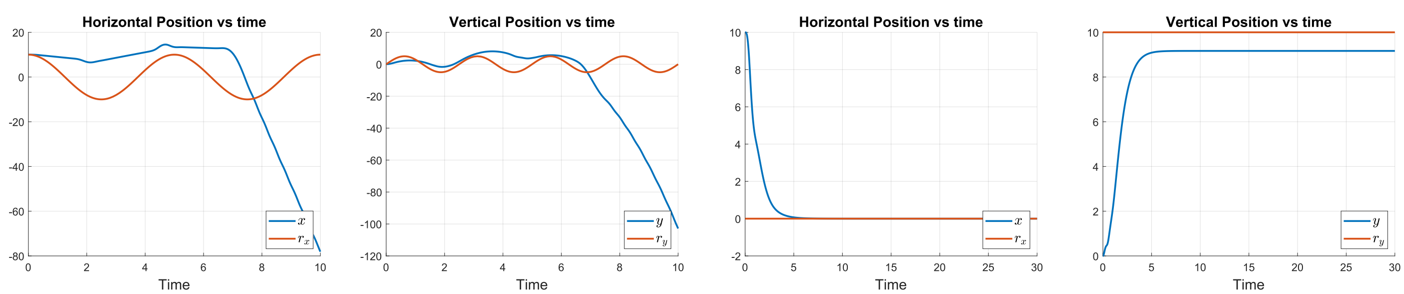
Fig. 7: Plant regulation for a Lemniscate of Gerono trajectory of amplitude 5 and period of 5 s. Divergence appears !

The linearization of the angle θ to 0 and the constraints imposed by the previously defined Q matrix limit the helicopter's speed and agility. As a result, the helicopter is forced to perform slower and more gradual movements, which can be particularly noticeable when sharp turns or sudden changes in direction are required, especially as the frequency increases. This limitation represents the first issue of the linear controller.

Now let's see how this linear regulator behave if the estimated parameters of the dynamics are off by a few percent.

2) Static linear controller with mismatched parameters:

Parameter	New Value	Error
m	4.8 kg	20%
d	0.11 kg/sec	10%
r	0.225 m	10%
J	0.1710 kg.m^2	20%



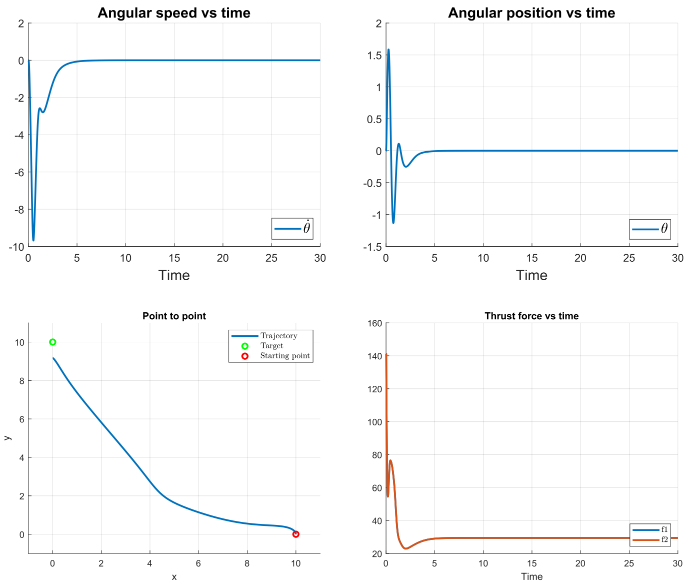


Fig. 8: Plant response for a point to point input using a LQR linear controller and wrong parameters estimation. Tracking is achieved with static error.

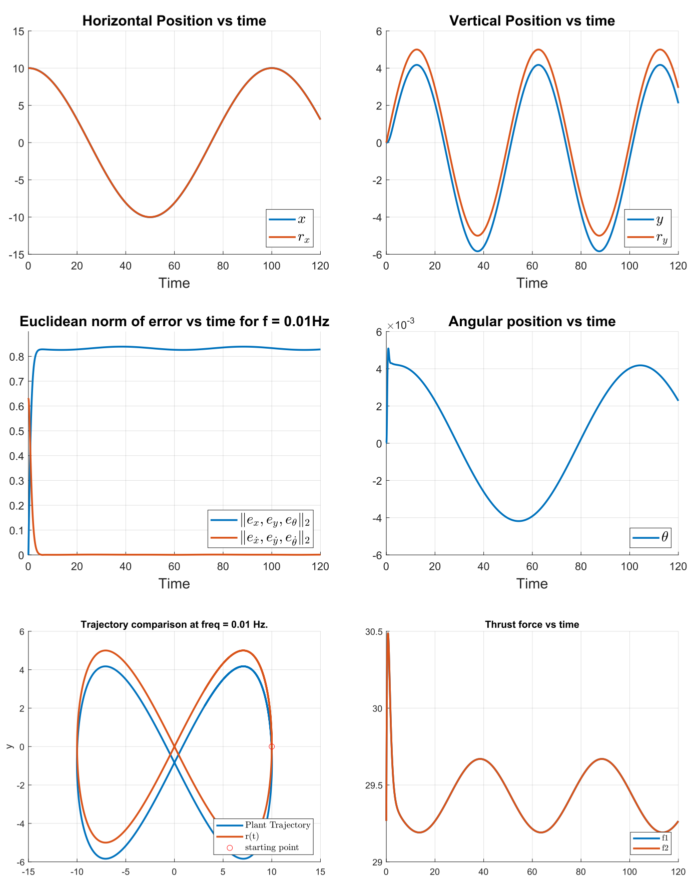


Fig. 9: Plant response for a Lemniscate of Gerono trajectory input using a LQR linear controller and wrong parameters estimation. Tracking is achieved with static error.

The controller exhibits a static error due to its inability to dynamically adapt to differences between the estimated model and the actual model. To address this issue, we will attempt to use an adaptive controller.

B. Linear Adaptive Control

In this part we design an linear adaptive controller using the previous model with estimated parameters as the reference model. The previous LQR controller (see Fig. 2) can be reduced to a dynamic $\dot{X}_m = A_m X_m + B_m r$ where

$$\begin{aligned} A_m &= [Ae - Bb^{-1}(\dot{r} - ax) + BK_{LQR}]X \\ Bm &= BK_{LQR}(r - x) \end{aligned} \tag{1}$$

We added a term $\alpha^T \phi$ to the control input to make sure that our gains don't diverge to compensate gravity and mismatched parameters. We took $\phi = 1$ as we know that gravity is a constant non linearity we want to compensate.

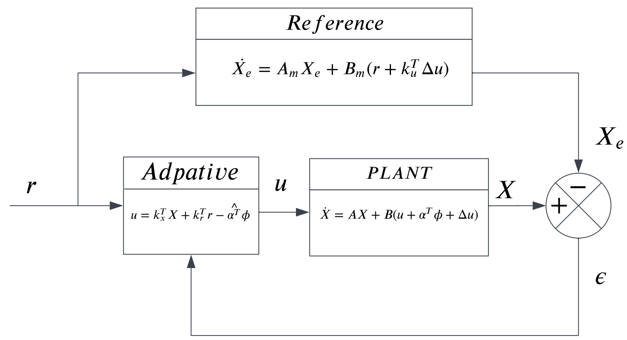


Fig. 10: Linear Adaptive Controller

Controller : $u = Sat({K_x}^T X + {K_r}^T r - \hat{\alpha}\phi)$
Update laws :

$$\begin{aligned} \dot{K}_x &= -\Gamma_x X e^T P B e \\ \dot{K}_r &= -\Gamma_r r e^T P B e \\ \dot{\alpha} &= \Gamma_\alpha \phi e^T P B e \\ \dot{K}_u &= \Gamma_u \Delta u B m \end{aligned} \tag{2}$$

P and K_{LQR} can been obtained from

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care(Ae,Be,Q);
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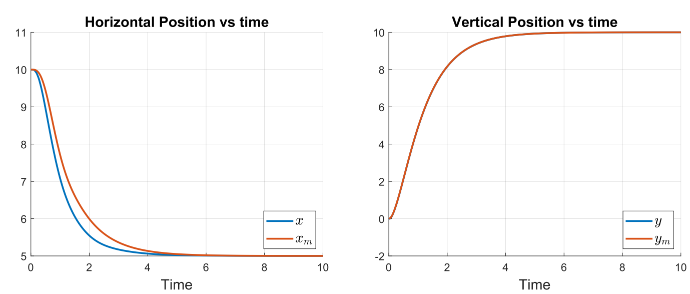
and $\Delta u = u - ({K_x}^T X + {K_r}^T r - \hat{\alpha}\phi)$, u being saturated at 10 times the baseline control.

The results presented below were obtained using selected gains from a previous post-transient simulation.



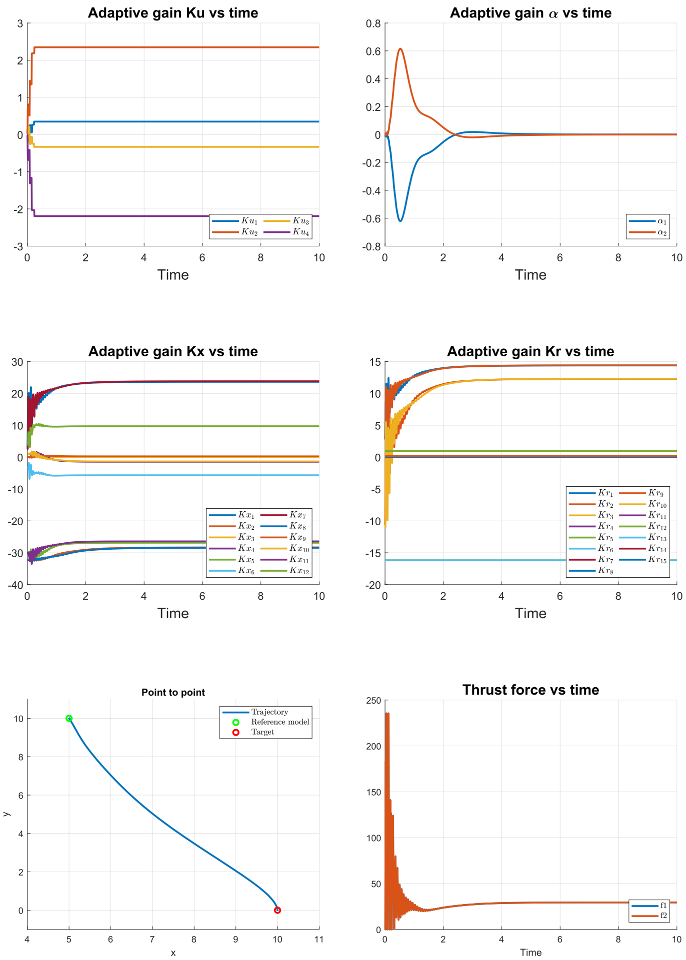
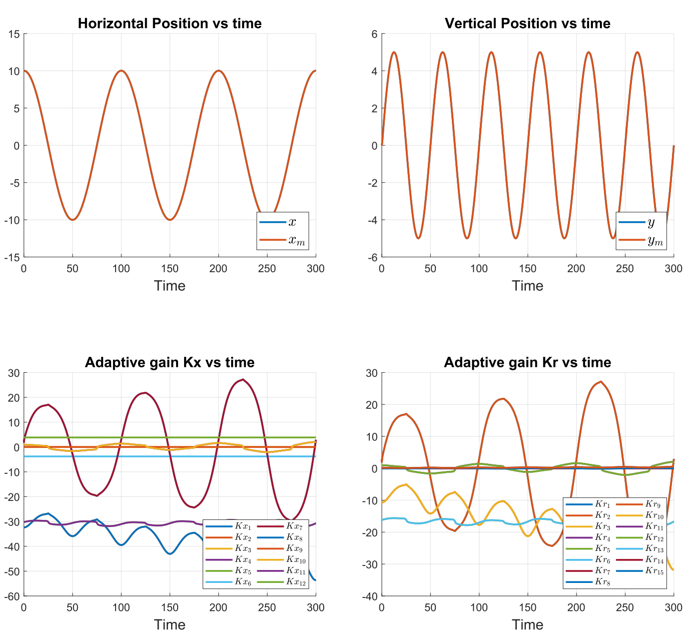


Fig. 11: Plant response for a Point to point input using a Linear Adaptive controller and wrong parameters estimation. All γ gains are set to 10.Tracking and compensation are achieved.



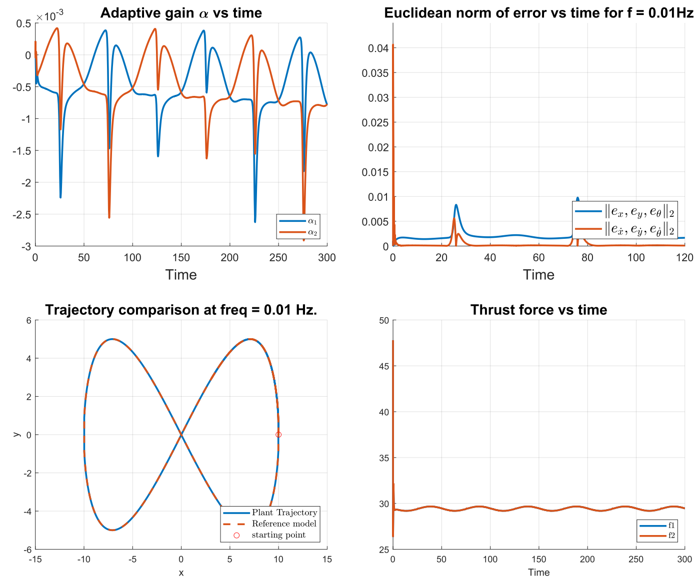


Fig. 12: Plant response for a Lemniscate of Gerono trajectory input of amplitude 10 and period of 100 s using a Linear Adaptive controller and wrong parameters estimation. All γ gains are set to 10. Tracking and compensation are achieved.

Summary

A linear controller performs adequately as long as the input demand for acceleration is not too high and the parameters are not significantly mismatched. However, we have observed that the use of a Model Reference Adaptive Controller (MRAC) solves the issue of mismatched parameters by dynamically adapting the system to the reference input. In the next step, we will approach the problem from a different perspective and suggest a Nonlinear Controller to investigate if it can achieve a higher attainable frequency while keeping a good tracking.

IV. Step 2 : Non Linear Controller

A. Static non linear controller with with matched parameters

Let's consider the sub-states,

$$\begin{aligned} X_q &= \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T \\ X_\theta &= \begin{bmatrix} \theta & \dot{\theta} \end{bmatrix}^T \end{aligned} \tag{3}$$

There exists a transformation of state that will provide full control of the position variables X_q if control over the orientation X_θ is relaxed.

$$q' = q + \lambda \begin{pmatrix} -sin(\theta) \\ cos(\theta) \end{pmatrix}$$

Where $q = \begin{pmatrix} x \\ y \end{pmatrix}$ and λ is a constant scalar. This transformation leads to the following new dynamics :

$$\begin{aligned} \dot{X}_q &= A_q X_q + B_q \Lambda_q (u + \alpha^T \phi) \\ \dot{X}_\theta &= A_\theta X_\theta + B_\theta \Lambda_\theta u \end{aligned} \tag{4}$$

$$A_q = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{d}{m} & 0 \\ 0 & 0 & 0 & -\frac{d}{m} \end{bmatrix} B_q = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

$$\Lambda_q = \begin{bmatrix} -\frac{\lambda r}{J} & \frac{\lambda r}{J} \\ \frac{1}{m} & \frac{1}{m} \end{bmatrix} \; A_\theta = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B_\theta = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \; \Lambda_\theta = \frac{r}{J}$$

Controller : $u = u_q + u_\theta$ with $u_q = \Lambda_q^{-1} B_q^{-1} u_L - \alpha^T \phi$ [1] where u_L and u_θ are linear controllers using feedback[2] and feedforward techniques such as in Section III-A.

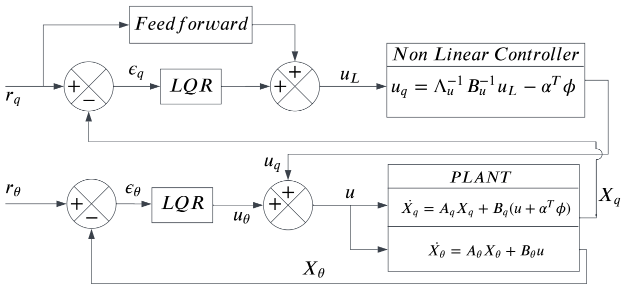


Fig. 13: Non Linear Controller

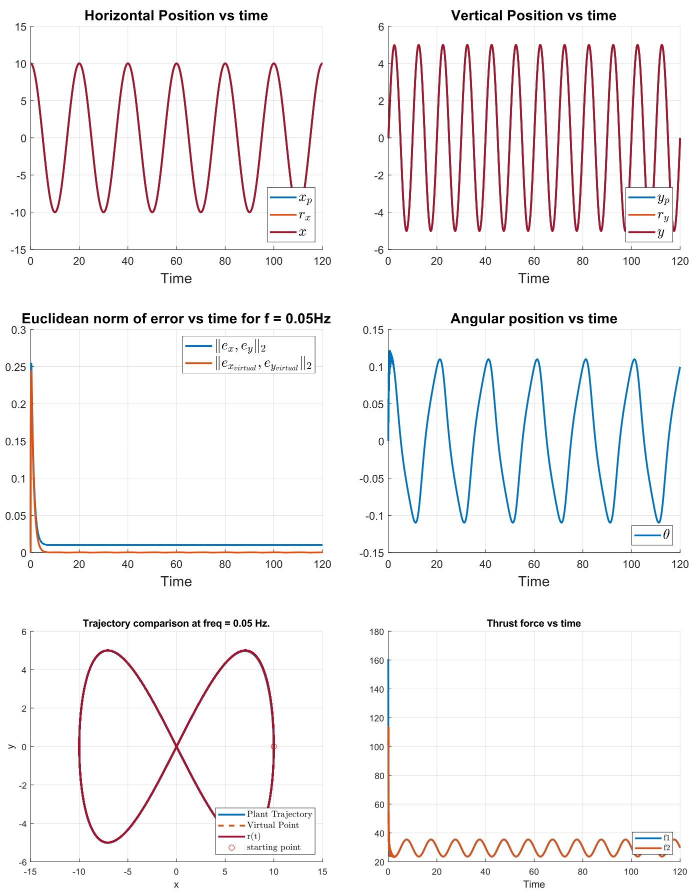


Fig. 14: Plant response for a Lemniscate of Gerono trajectory input of amplitude 10 and period of 20 s using a Nonlinear Controller. Tracking is achieved with a lower error than the Linear Controller (Fig. 5).

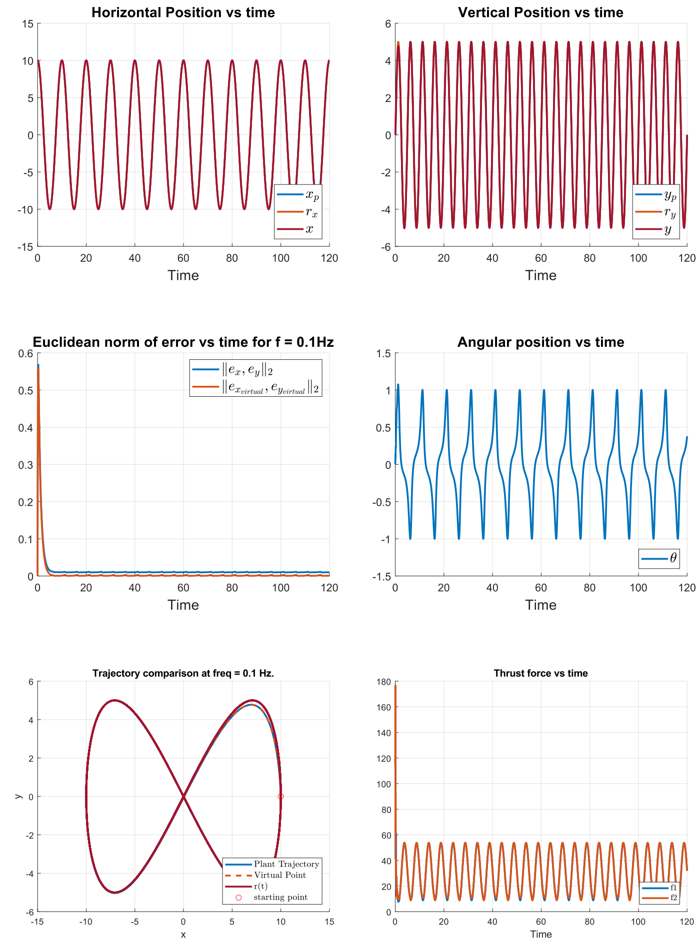
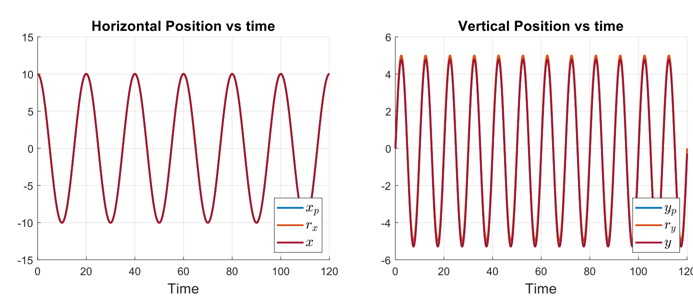


Fig. 15: Plant response for a Lemniscate of Gerono trajectory input of amplitude 10 and period of 10 s using a Nonlinear Controller. Tracking is achieved with a significant lower error than the Linear Controller (Fig. 6). Therefore, the static Nonlinear Controller better meets the performance objectives than the static Linear Controller.

Now, let's examine how this nonlinear controller behaves if the estimated parameters of the dynamics are off by a few percent, as in Section III-A2

B. Static non linear controller with with mismatched parameters



[1] Generalized inverse $B_q^{-1} = \begin{bmatrix} 0 & 0 & cos(\theta) & sin(\theta) \\ 0 & 0 & -sin(\theta) & cos(\theta) \end{bmatrix}$

[2] The reference r_θ is chosen to be constantly zero.

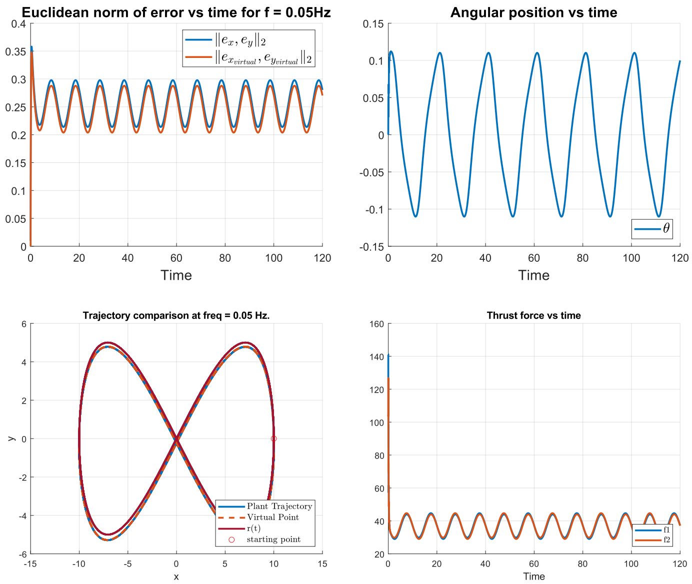


Fig. 16: Plant response for a Lemniscate of Gerono trajectory input of amplitude 10 and period of 20 s using a Nonlinear Controller with wrong parameters estimation. Tracking is achieved with static error.

Summary

A static non-linear controller can better meet performance objectives than a static linear controller. However, if parameter estimation is off, the performance deteriorates since the static controller cannot dynamically adapt to model mismatch. In the next step, we will attempt to address this issue by using an adaptive controller.

V. Step 3 : Non Linear MRAC

By building upon the previous non-linear controller, it is possible to expand it and create a reference model that can be used to design a non-linear adaptive controller. As previously, the objective is to compensate mismatched parameters.
Let's keep working with the sub-states dynamics:

$$\begin{aligned} \dot{X}_q &= A_q X_q + B_q \Lambda_q (u + \alpha^T \phi) \\ \dot{X}_\theta &= A_\theta X_\theta + B_\theta \Lambda_\theta u \end{aligned} \tag{5}$$

Controller and Update law are the same as the one developed in set of equations (2). Construction of A_m and B_m is based on the previous non-linear controller (see Fig. 13).

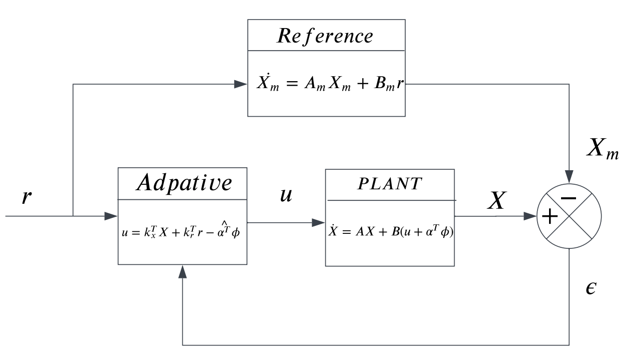


Fig. 17: Non Linear Adaptive Controller

The matching conditions are:

$$\begin{aligned} BK_x^* &= Am - A \\ BK_r^* &= Bm \\ \hat{\alpha}^* &= \alpha \end{aligned} \tag{6}$$

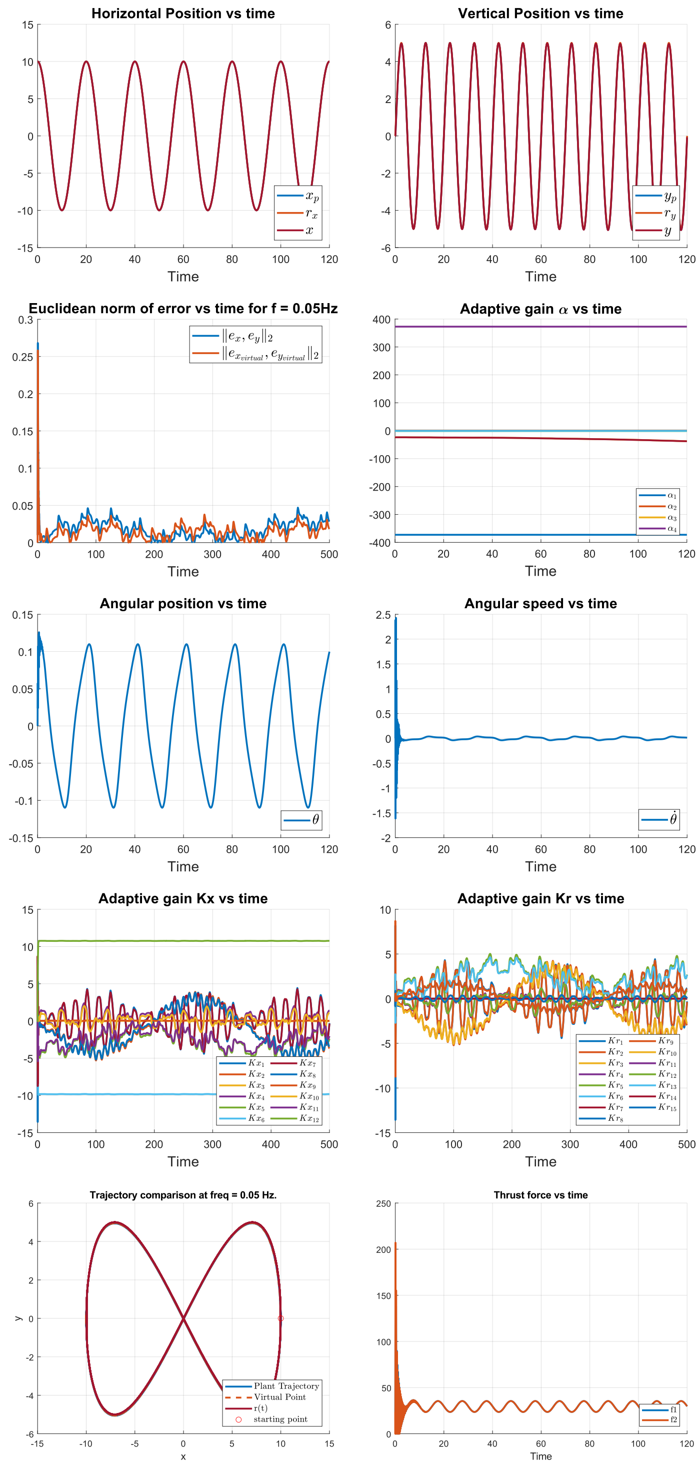


Fig. 18: Plant response for a Lemniscate of Gerono trajectory input of amplitude 10 and period of 20 s using a Nonlinear Controller. Tracking is achieved with a lower error than the Non Linear Controller. All gains are bounded or converge. (Fig. 5).

VI. Conclusion

In conclusion, the Planar Bi-Rotor Helicopter project presented a challenging task of designing an adaptive control system for a bi-rotor helicopter that can meet performance objectives in the presence of model mismatch and uncertainty. The project started with the linearization of the equations of motion about hover and the design of a linear feedback controller that stabilizes the system and meets the performance specifications.

Then, we introduced wrong parameters estimations by modifying some of the system's parameters and comparing the outcomes under a traditional linear controller versus an adaptive controller. The adaptive controller was designed to estimate the unknown model parameters and update the control gains in real-time, improving the system's performance.

The next step involved designing a non-linear controller for the state position and orientation. Results showed that it performed better than the linear controller and was able to track trajectories for higher frequencies with lower error.

In the last step, we then introduced wrong parameters estimations and augmented the static non linear controller to an adaptive non linear controller. Simulation results showed that the adaptive controller was able to track reference signals with better accuracy and reduced tracking error by adapting to parameters estimation.

Overall, this project demonstrated the importance of adaptive control in dealing with model uncertainty and parameter variations and highlighted the benefits of designing control systems that can adapt to changing conditions.